



THE ARITHMETIC OF EQUATIONS

section 9.1

Silk, one of the most beautiful and luxurious of all fabrics, is spun from the cocoons of tiny silkworms. Silkworms have the unique ability to transform the leaves of mulberry trees into silk thread, which they use to weave their cocoons.

More than 3000 cocoons are needed to produce enough silk to make just one elegant Japanese kimono. How do chemists calculate the amount of reactants and products in chemical reactions?

Using Everyday Equations

Nearly everything you use is manufactured from chemicals—soaps, shampoos and conditioners, cassette tapes, cosmetics, medicines, and clothes. Obviously, for the manufacturer to make a profit, the cost of making any of these items cannot exceed the money paid for them. Therefore, the chemical processes used in manufacturing must be carried out economically. This is where balanced chemical equations help.

Equations are the recipes that tell chemists what amounts of reactants to mix and what amounts of products to expect. You can determine the quantities of reactants and products in a reaction from the balanced equation. When you know the quantity of one substance in a reaction, you can calculate the quantity of any other substance consumed or created in the reaction. Quantity usually means the amount of a substance expressed in grams or moles. But quantity could just as well be in liters, tons, or molecules. Can you name some other units you might use to measure the amount of matter?

The calculation of quantities in chemical reactions is a subject of chemistry called **stoichiometry**. Calculations using balanced equations are called stoichiometric calculations. For chemists, stoichiometry is a form of bookkeeping.

When you bake cookies, you probably use a recipe. A cookie recipe tells you the amounts of ingredients to mix together to make a certain number of cookies. If you need a larger number of cookies than the recipe provides, the amounts of ingredients can be doubled or tripled. In a way, a cookie recipe provides the same kind of information that a balanced chemical equation does. The ingredients are the reactants; the cookies are the products.

objectives

- ▶ Calculate the amount of reactants required or product formed in a nonchemical process
- ▶ Interpret balanced chemical equations in terms of interacting moles, representative particles, masses, and gas volume at STP

key term

- ▶ stoichiometry

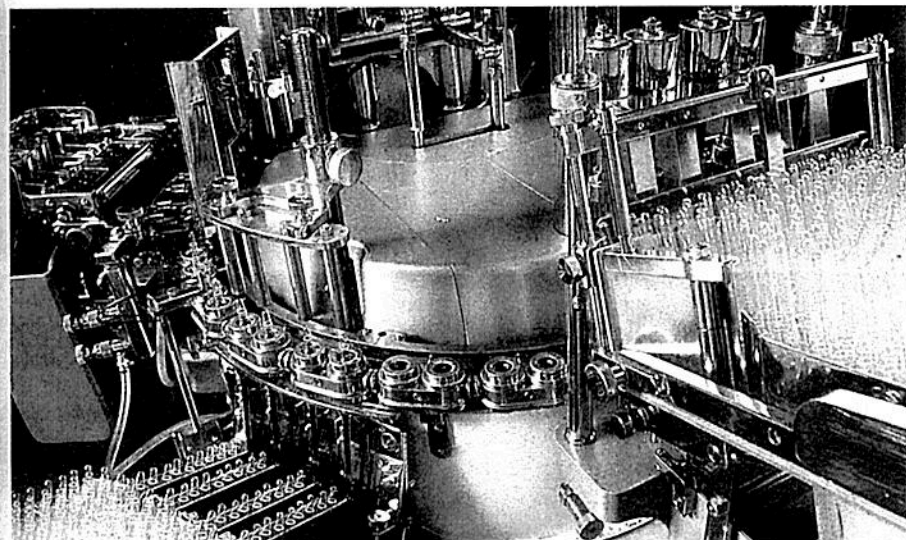
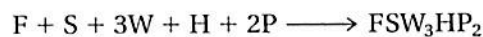


Figure 9.1

Just like cooking, manufacturing requires specific amounts of ingredients to get a certain number of products.

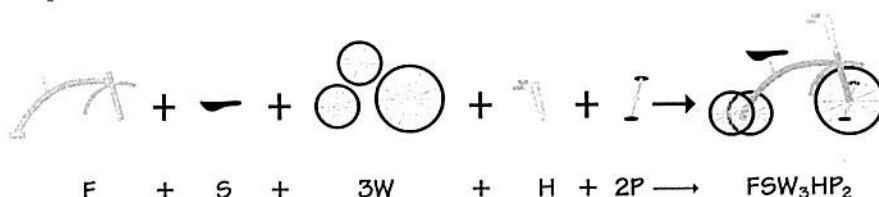
Here is another example, this time from the business world rather than from the world of cooking. Imagine you are in charge of manufacturing for the Tiny Tyke Tricycle Company. The business plan for Tiny Tyke requires the production of 128 custom-made tricycles each day. One of your responsibilities is to be sure that there are enough parts available at the start of each day to make these tricycles. To simplify this discussion, assume that the major components of the tricycle are the frame (F), the seat (S), the wheels (W), the handlebars (H), and the pedals (P). The finished tricycle has a "formula" of FSW_3HP_2 . The balanced equation for the production of a tricycle is



This equation gives you the "recipe" to make a single tricycle: Making a tricycle requires one frame, one seat, three wheels, one handlebar, and two pedals.

Figure 9.2

A balanced equation can be thought of as a recipe. In the equation shown here, the tricycle parts are the reactants and the assembled tricycle is the product. How many pedals are needed to make four tricycles?



Sample Problem 9-1

In a five-day workweek, Tiny Tyke is scheduled to make 640 tricycles. How many wheels should be in the plant on Monday morning to make these tricycles?

1. ANALYZE List the knowns and the unknown.

Knowns:

- number of tricycles = 640 tricycles
- $1 \text{ FSW}_3\text{HP}_2 = 3 \text{ W}$ (from balanced equation)

Unknown:

- number of wheels = ? wheels

Use the conversion factor $\frac{3 \text{ W}}{1 \text{ FSW}_3\text{HP}_2}$ to calculate the unknown.

2. CALCULATE Solve for the unknown.

$$640 \text{ FSW}_3\text{HP}_2 \times \frac{3 \text{ W}}{1 \text{ FSW}_3\text{HP}_2} = 1920 \text{ W}$$

3. EVALUATE Does the result make sense?

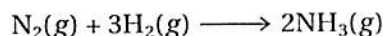
If 3 wheels are required for each tricycle, and a total of more than 600 tricycles are being made, then a number of wheels in excess of 1800 is a logical answer. The unit of the known cancels with the unit in the denominator of the conversion factor, and the answer is in the unit of the unknown. The conversion factor is exact and does not affect the rounding of the answer.

Practice Problems

1. How many tricycle seats, wheels, and pedals are needed to make 288 tricycles?
2. Write an equation that gives your own "recipe" for making a puppet or a piece of furniture.




Interpreting Chemical Equations

As you may recall from Chapter 7, ammonia is widely used as a fertilizer. Ammonia is produced industrially by the reaction of nitrogen with hydrogen.



What kinds of information can be derived from this equation?

- Particles** One molecule of nitrogen reacts with three molecules of hydrogen to produce two molecules of ammonia. Nitrogen and hydrogen will always react to form ammonia in this 1:3:2 ratio of molecules. So if you could make 10 molecules of nitrogen react with 30 molecules of hydrogen, you would expect to get 20 molecules of ammonia. Of course, it is not possible to count such small numbers of molecules and allow them to react. You could, however, take Avogadro's number of nitrogen molecules and make them react with three times Avogadro's number of hydrogen molecules. This would be the same 1:3 ratio of molecules of reactants. The reaction would form two times Avogadro's number of ammonia molecules.
- Moles** You know that Avogadro's number of representative particles is one mole of a substance. On the basis of the particle interpretation you just read, the equation tells you the number of moles of reactants and products. One mole of nitrogen molecules reacts with three moles of hydrogen molecules to form two moles of ammonia molecules. The coefficients of a balanced chemical equation indicate the relative numbers of moles of reactants and products in a chemical reaction. This is the most important information that a balanced chemical equation provides. Using this information, you can calculate the amounts of reactants and products. Does the number of moles of reactants equal the number of moles of product in this reaction?

$\text{N}_2(\text{g})$	+	$3\text{H}_2(\text{g})$	\longrightarrow	$2\text{NH}_3(\text{g})$
	+		\longrightarrow	
2 atoms N	+	6 atoms H	\longrightarrow	2 atoms N and 6 atoms H
1 molecule N_2	+	3 molecules H_2	\longrightarrow	2 molecules NH_3
10 molecules N_2	+	30 molecules H_2	\longrightarrow	20 molecules NH_3
$1 \times (6.02 \times 10^{23})$ molecules N_2	+	$3 \times (6.02 \times 10^{23})$ molecules H_2	\longrightarrow	$2 \times (6.02 \times 10^{23})$ molecules NH_3
1 mol N_2	+	3 mol H_2	\longrightarrow	2 mol NH_3
28 g N_2	+	$3 \times 2 \text{ g H}_2$	\longrightarrow	$2 \times 17 \text{ g NH}_3$
		34 g reactants	\longrightarrow	34 g products
Assume STP		22.4 L	22.4 L 22.4 L	22.4 L 22.4 L
	+			
22.4 L N_2		67.2 L H_2		44.8 L NH_3

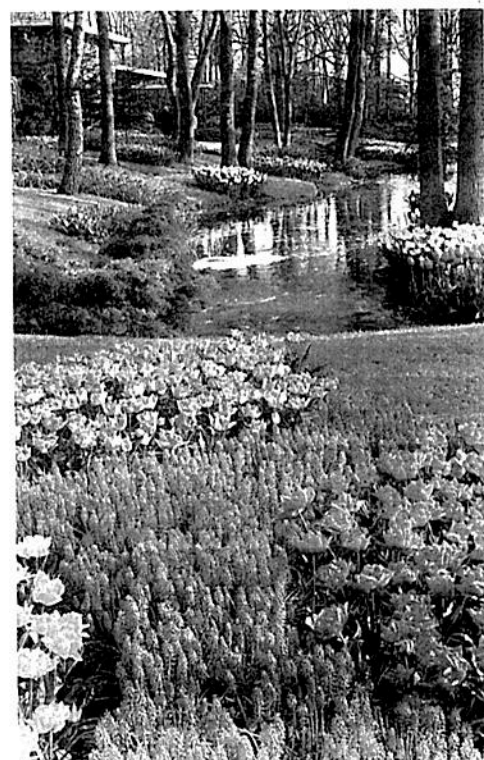


Figure 9.3

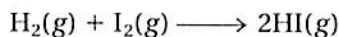
Gardeners use ammonium salts as fertilizer. The nitrogen in these salts is essential to plant growth.

Figure 9.4

The balanced chemical equation for the formation of ammonia can be interpreted in several ways. How many molecules of NH_3 could be made from 5 molecules N_2 and 15 molecules H_2 ?

3. **Mass** A balanced chemical equation must obey the law of conservation of mass. This law states that mass can be neither created nor destroyed in an ordinary chemical or physical process. The mole interpretation supports this requirement. Remember that mass is related to the number of atoms in the chemical equation through moles. The mass of 1 mol of nitrogen (28.0 g) plus the mass of 3 mol of hydrogen (6.0 g) does equal the mass of 2 mol of ammonia (34.0 g). So although the number of moles of reactants does not equal the number of moles of product(s), the total number of grams of reactants does equal the total number of grams of product(s). Mass is conserved.
4. **Volume** If you assume standard temperature and pressure, the equation also tells you about the volumes of gases. Recall that 1 mol of any gas at STP occupies a volume of 22.4 L. It follows that 22.4 L of nitrogen reacts with 67.2 L (3×22.4 L) of hydrogen to form 44.8 L (2×22.4 L) of ammonia.

Look at Figure 9.4 on the previous page. Do you see that mass and atoms are conserved in this chemical reaction? Mass and atoms are conserved in every chemical reaction. The mass of the reactants equals the mass of the products. The number of atoms of each reactant equals the number of atoms of that reactant in the product(s). Unlike mass and atoms, however, molecules, formula units, moles, and volumes of gases will not necessarily be conserved—although they may. Consider, for example, the formation of hydrogen iodide.



In this reaction, molecules, moles, and volume are all conserved. But in the majority of chemical reactions (including the reaction for the formation of ammonia), they are not. Only mass and atoms are conserved in every chemical reaction.



Figure 9.5

Hydrogen sulfide (H_2S) smells like rotten eggs. It escapes from the ground in volcanic areas.

Sample Problem 9-2

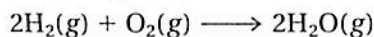
Hydrogen sulfide, a foul-smelling gas, is found in nature in volcanic areas. The balanced chemical equation for the burning of hydrogen sulfide is given below. Interpret this equation in terms of the interaction of the following three relative quantities.

- a. number of representative particles
- b. number of moles
- c. masses of reactants and products



Practice Problems

3. Interpret the equation for the formation of water from its elements in terms of numbers of molecules and moles and volumes of gases at STP.



1. **ANALYZE** Plan a problem-solving strategy.

- a. The coefficients in the balanced equation give the relative number of molecules of reactants and products.
- b. The coefficients in the balanced equation give the relative number of moles of reactants and products.
- c. A balanced chemical equation obeys the law of conservation of mass. The sum of the masses of the reactants must equal the sum of the masses of the products.

Sample Problem 9-2 (cont.)

 2. **SOLVE** Apply the problem-solving strategy.

- 2 molecules H_2S react with 3 molecules O_2 to form 2 molecules SO_2 and 2 molecules H_2O .
- 2 mol H_2S react with 3 mol O_2 to produce 2 mol SO_2 and 2 mol H_2O .
- Multiply the number of moles of each reactant and product by its molar mass: $2 \text{ mol } \text{H}_2\text{S} + 3 \text{ mol } \text{O}_2 \rightarrow 2 \text{ mol } \text{SO}_2 + 2 \text{ mol } \text{H}_2\text{O}$.

$$\left(2 \text{ mol} \times 34.1 \frac{\text{g}}{\text{mol}}\right) + \left(3 \text{ mol} \times 32.0 \frac{\text{g}}{\text{mol}}\right) \longrightarrow$$

$$\left(2 \text{ mol} \times 64.1 \frac{\text{g}}{\text{mol}}\right) + \left(2 \text{ mol} \times 18.0 \frac{\text{g}}{\text{mol}}\right)$$

$$68.2 \text{ g } \text{H}_2\text{S} + 96.0 \text{ g } \text{O}_2 \longrightarrow 128.2 \text{ g } \text{SO}_2 + 36.0 \text{ g } \text{H}_2\text{O}$$

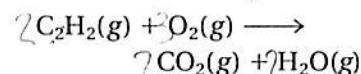
$$164.2 \text{ g} = 164.2 \text{ g}$$

 3. **EVALUATE** Do the results make sense?

Because all the substances in this reaction are molecular, the mole ratio of reactants and products equals the molecular ratio of reactants and products (2:3:2:2). The sum of the masses of the reactants equals the sum of the masses of the products.

Practice Problems (cont.)

4. Balance the equation for the combustion of acetylene:



Interpret the equation in terms of relative numbers of moles, volumes of gas at STP, and masses of reactants and products.

Chem ASAP!

Problem-Solving 4

Solve Problem 4 with the help of an interactive guided tutorial.



section review 9.1

5. Your school club has "adopted" a local nursing home and provides welcoming packages to new residents. Each welcoming package contains a toothbrush (B), three washcloths (W), a hand mirror (M), two decks of cards (C), and four small bottles of skin lotion (L).

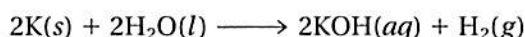
- Write a balanced equation for preparing a welcoming package ($\text{BW}_3\text{MC}_2\text{L}_4$).
- Calculate the number of each item needed for 45 packages.

6. Balance this equation:
- $\text{C}_2\text{H}_5\text{OH}(\text{l}) + \text{O}_2(\text{g}) \longrightarrow \text{CO}_2(\text{g}) + \text{H}_2\text{O}(\text{g})$
- .

- Interpret the equation in terms of numbers of molecules and moles.
- Show that the balanced equation obeys the law of conservation of mass.

7. Explain this statement: "Mass and atoms are conserved in every chemical reaction, but moles will not necessarily be conserved."

8. Interpret the following equation in terms of relative numbers of representative particles, numbers of moles, and masses of reactants and products.



Chem ASAP! Assessment 9.1 Check your understanding of the important ideas and concepts in Section 9.1.



objectives

- Construct mole ratios from balanced chemical equations and apply these ratios in mole-mole stoichiometric calculations
- Calculate stoichiometric quantities from balanced chemical equations using units of moles, mass, representative particles, and volumes of gases at STP

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Simulation 6

Strengthen your analytical skills by solving stoichiometric problems.



Air bags inflate almost instantaneously upon impact. The effectiveness of air bags is based on the rapid conversion of a small mass of sodium azide into a large volume of gas. How can stoichiometry be used to calculate the volume of gas produced in this reaction?

Mole-Mole Calculations

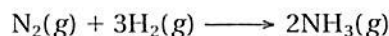
As you just learned, a balanced chemical equation provides a wealth of quantitative information relating representative particles (atoms, molecules, formula units), moles of substances, and masses. Most important, a balanced chemical equation is essential for all calculations involving amounts of reactants and products: If you know the number of moles of one substance, the balanced chemical equation allows you to determine the number of moles of all other substances in the reaction.



Figure 9.6

Manufacturing plants produce ammonia by combining nitrogen with hydrogen. Ammonia is used in cleaning products, fertilizers, and in the manufacture of other chemicals.

Look again at the production of ammonia from nitrogen and hydrogen. The balanced equation for the reaction is



The most important interpretation of this equation is that 1 mol of nitrogen reacts with 3 mol of hydrogen to form 2 mol of ammonia. With this interpretation, you can relate moles of reactants to moles of product. The coefficients from the balanced equation are used to write conversion factors called mole ratios. The mole ratios are used to calculate the number of moles of product from a given number of moles of reactant or to calculate the number of moles of reactant from a given number of moles of product. Three of the mole ratios for this equation are

$$\frac{1 \text{ mol N}_2}{3 \text{ mol H}_2} \quad \frac{2 \text{ mol NH}_3}{1 \text{ mol N}_2} \quad \frac{3 \text{ mol H}_2}{2 \text{ mol NH}_3}$$

What are the other three mole ratios?

Practice Problems

9. This equation shows the formation of aluminum oxide.
- $$4\text{Al(s)} + 3\text{O}_2\text{(g)} \longrightarrow 2\text{Al}_2\text{O}_3\text{(s)}$$
- Write the six mole ratios that can be derived from this equation.
 - How many moles of aluminum are needed to form 3.7 mol Al_2O_3 ?
10. According to the equation in Problem 9:
- How many moles of oxygen are required to react completely with 14.8 mol Al?
 - How many moles of Al_2O_3 are formed when 0.78 mol O_2 reacts with aluminum?

Sample Problem 9-3

How many moles of ammonia are produced when 0.60 mol of nitrogen reacts with hydrogen?

1. ANALYZE List the known and the unknown.

Known:

- moles of nitrogen = 0.60 mol N_2

Unknown:

- moles of ammonia = ? mol NH_3

The conversion is mol $\text{N}_2 \rightarrow$ mol NH_3 . According to the balanced equation, 1 mol N_2 combines with 3 mol H_2 to produce 2 mol NH_3 . To determine the number of moles of NH_3 , the given quantity of N_2 is multiplied by the form of the mole ratio from the balanced equation that allows the given unit to cancel.

2. CALCULATE Solve for the unknown.

$$0.60 \text{ mol } \text{N}_2 \times \frac{2 \text{ mol } \text{NH}_3}{1 \text{ mol } \text{N}_2} = 1.2 \text{ mol } \text{NH}_3$$

3. EVALUATE Does the result make sense?

The balanced chemical equation shows that two moles of ammonia are produced for each mole of nitrogen reacted. Note that mole ratios from balanced equations are considered to be exact (defined numbers). They do not enter into the determination of significant figures in the answer.



Figure 9.7

To determine the number of moles in a sample of a compound, first measure the mass of the sample. Then use the molar mass to calculate the number of moles in that mass.

In the mole ratio below, W is the unknown quantity. The values of a and b are the coefficients from the balanced equation. Thus a general solution for a mole-mole problem, such as Sample Problem 9-3, is given by

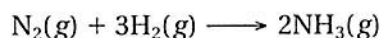
$$\underbrace{x \text{ mol } G}_{\text{Given}} \times \underbrace{\frac{b \text{ mol } W}{a \text{ mol } G}}_{\text{Mole ratio}} = \underbrace{\frac{xb}{a} \text{ mol } W}_{\text{Calculated}}$$

Mass-Mass Calculations

No laboratory balance can measure substances directly in moles. Instead, as is shown in Figure 9.7, the amount of a substance is usually determined by measuring its mass in grams. From the mass of a reactant or product, the mass of any other reactant or product in a given chemical equation can be calculated. The mole interpretation of a balanced equation is the basis for this conversion. If the given sample is measured in grams, the mass can be converted to moles by using the molar mass. Then the mole ratio from the balanced equation can be used to calculate the number of moles of the unknown. If it is the mass of the unknown that needs to be determined, the number of moles of the unknown can be multiplied by the molar mass. As in mole-mole calculations, the unknown can be either a reactant or a product.

Sample Problem 9-4

Calculate the number of grams of NH_3 produced by the reaction of 5.40 g of hydrogen with an excess of nitrogen. The balanced equation is



1. ANALYZE List the knowns and the unknown.

Knowns:

- mass of hydrogen = 5.40 g H_2
- 3 mol H_2 = 2 mol NH_3 (from balanced equation)
- 1 mol H_2 = 2.0 g H_2 (molar mass)
- 1 mol NH_3 = 17.0 g NH_3 (molar mass)

Unknown:

- mass of ammonia = ? g NH_3

The mass in grams of hydrogen will be used to find the mass in grams of ammonia: g $\text{H}_2 \longrightarrow$ g NH_3 .

The coefficients in the balanced equation show that 3 mol H_2 reacts with 1 mol N_2 to produce 2 mol NH_3 . The following calculations need to be done:



2. CALCULATE Solve for the unknown.

Convert the given (5.40 g H_2) to moles by using the molar mass of hydrogen.

$$5.40 \text{ g H}_2 \times \frac{1 \text{ mol H}_2}{2.0 \text{ g H}_2} = 2.7 \text{ mol H}_2$$

Use the mole ratio from the balanced equation to calculate the number of moles of NH_3 .

$$2.7 \text{ mol H}_2 \times \frac{2 \text{ mol NH}_3}{3 \text{ mol H}_2} = 1.8 \text{ mol NH}_3$$

Convert the moles of NH_3 to grams of NH_3 by using the molar mass of ammonia.

$$1.8 \text{ mol NH}_3 \times \frac{17.0 \text{ g NH}_3}{1 \text{ mol NH}_3} = 31 \text{ g NH}_3$$

This series of calculations can be combined:

$$5.40 \text{ g H}_2 \times \frac{1 \text{ mol H}_2}{2.0 \text{ g H}_2} \times \frac{2 \text{ mol NH}_3}{3 \text{ mol H}_2} \times \frac{17.0 \text{ g NH}_3}{1 \text{ mol NH}_3} = 31 \text{ g NH}_3$$

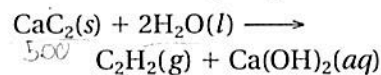
Given quantity	Change given unit to moles	Mole ratio	Change moles to grams
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3. EVALUATE Does the result make sense?

Because there are three conversion factors involved in this solution, it is more difficult to estimate an answer. However, because the molar mass of NH_3 is substantially greater than the molar mass of H_2 , the answer should have a larger mass than the given mass. The answer should have two significant figures.

Practice Problems

11. Acetylene gas (C_2H_2) is produced by adding water to calcium carbide (CaC_2).



How many grams of acetylene are produced by adding water to 5.00 g CaC_2 ?

12. Using the same equation, determine how many moles of CaC_2 are needed to react completely with 49.0 g H_2O .

Chem ASAP!

Problem-Solving 11

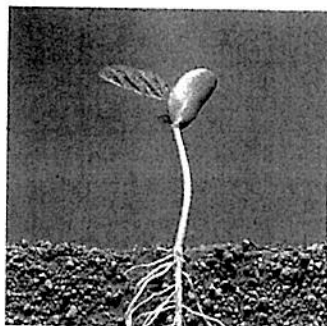
Solve Problem 11 with the help of an interactive guided tutorial.



LINK TO AGRICULTURE

Ammonia in the Nitrogen Cycle

Ammonia is part of the nitrogen cycle in nature. Earth's atmosphere contains 0.01 parts per million of ammonia, and small



amounts of ammonia occur in volcanic gases. Most ammonia cycles through the living world without returning to the atmosphere. Ammonia plays a role in several stages of the nitrogen cycle. Nitrogen-fixing bacteria form nodules, or swellings, on the roots of plants in the legume family, such as beans and clover plants. Here the bacteria change atmospheric nitrogen into ammonia molecules or ammonium ions. Other bacteria break down the nitrogenous material in dead plants and animals into ammonia molecules. Certain soil bacteria oxidize these molecules into nitrate ions, the form readily absorbed by plant roots. When a plant dies, this cycle begins again.

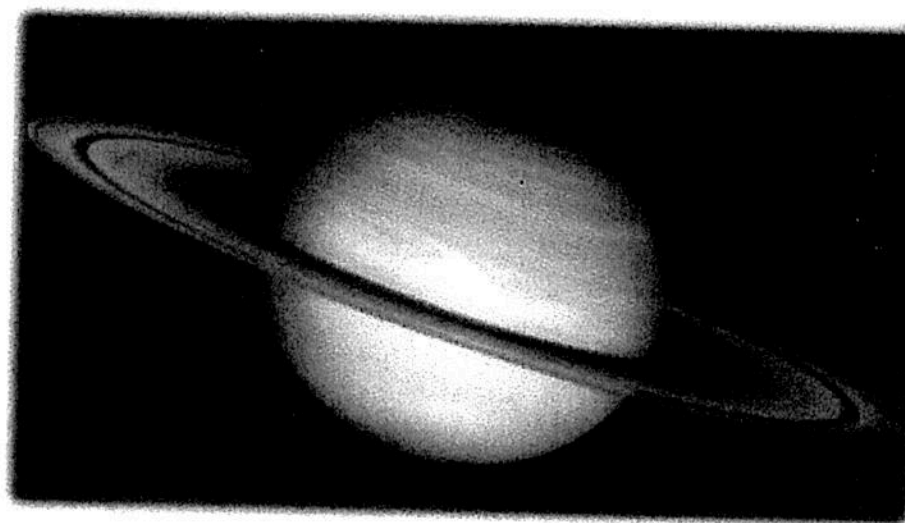


Figure 9.8

In this Hubble Space Telescope image, clouds of condensed ammonia are visible covering the surface of Saturn.

If the law of conservation of mass is true, how is it possible to make 31 g NH_3 from only 5.40 g H_2 ? Looking back at the equation for the reaction, you will see that hydrogen is not the only reactant. Another reactant, nitrogen, is also involved. If you were to calculate the number of grams of nitrogen needed to produce 31 g NH_3 and then compare the total masses of reactants and products, you would have an answer to this question. Go ahead and try it!

You can see from Sample Problem 9-4 that mass-mass problems can be solved in basically the same way as mole-mole problems. Figure 9.9 reviews the steps for the mass-mass conversion of any given mass (G) and any wanted mass (W).

1. The mass G is changed to moles of G (mass $G \longrightarrow \text{mol } G$) by using the molar mass of G .

$$\text{mass } G \times \frac{1 \text{ mol } G}{\text{molar mass } G} = \text{mol } G$$

2. The moles of G are changed to moles of W ($\text{mol } G \longrightarrow \text{mol } W$) by using the mole ratio from the balanced equation.

$$\text{mol } G \times \frac{b \text{ mol } W}{a \text{ mol } G} = \text{mol } W$$

3. The moles of W are changed to grams of W ($\text{mol } W \longrightarrow \text{mass } W$) by using the molar mass of W .

$$\text{mol } W \times \frac{\text{molar mass } W}{1 \text{ mol } W} = \text{mass } W$$

Figure 9.9 also shows the steps for doing mole-mass and mass-mole stoichiometric calculations. For a mole-mass problem, the first conversion (from mass to moles) is skipped. For a mass-mole problem, the last conversion (from moles to mass) is skipped. You can use parts of the three-step process shown in Figure 9.9 as they are appropriate to the problem you are solving.

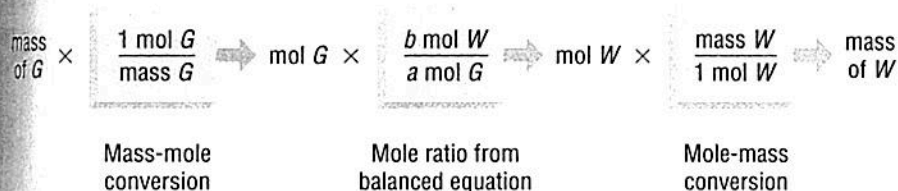
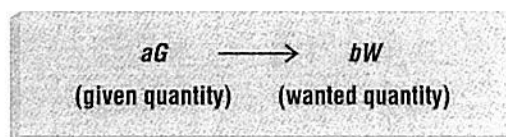


Figure 9.9

This general solution diagram indicates the steps necessary to solve a mass-mass stoichiometry problem: convert mass to moles, use the mole ratio, and then convert moles to mass. Is the given always a reactant?

Other Stoichiometric Calculations

As you already know, a balanced chemical equation indicates the relative number of moles of reactants and products. From this foundation, stoichiometric calculations can be expanded to include any unit of measurement that is related to the mole. The given quantity can be expressed in numbers of representative particles, units of mass, or volumes of gases at STP. The problems can include mass-volume, volume-volume, and particle-mass calculations. In any of these problems, the given quantity is first converted to moles. Then the mole ratio from the balanced equation is used to calculate the number of moles of the wanted substance. Once this has been determined, the moles are converted to any other unit of measurement related to the unit mole, as the problem requires. Figure 9.10 summarizes these steps for a typical stoichiometric problem.

Figure 9.10

With your knowledge of conversion factors and this problem-solving approach you can solve a variety of stoichiometric problems. What conversion factor is used to convert moles to representative particles?

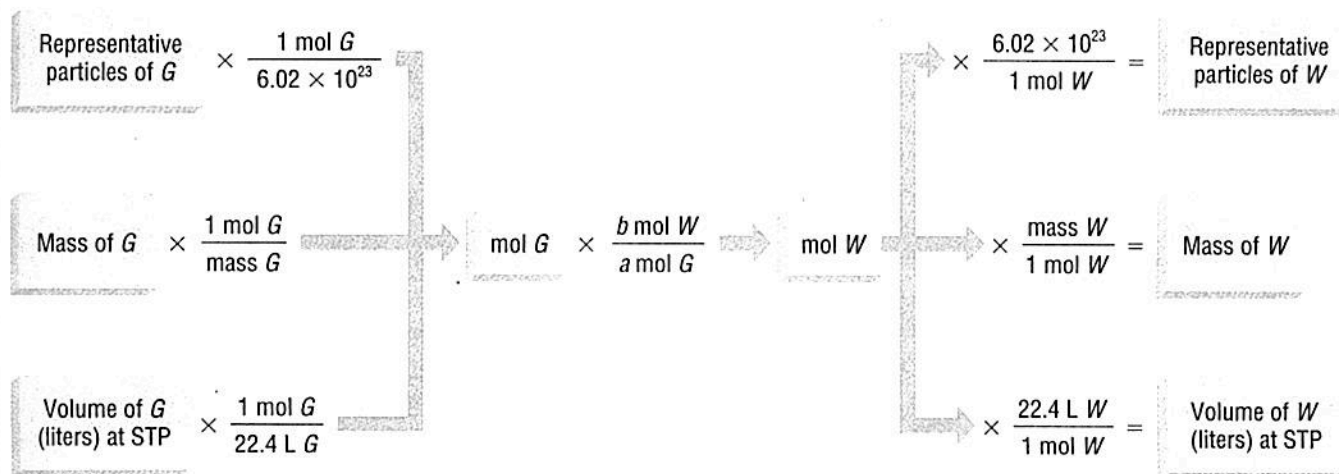
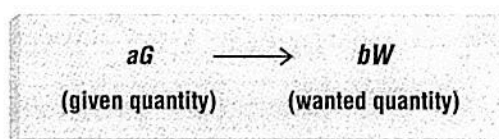
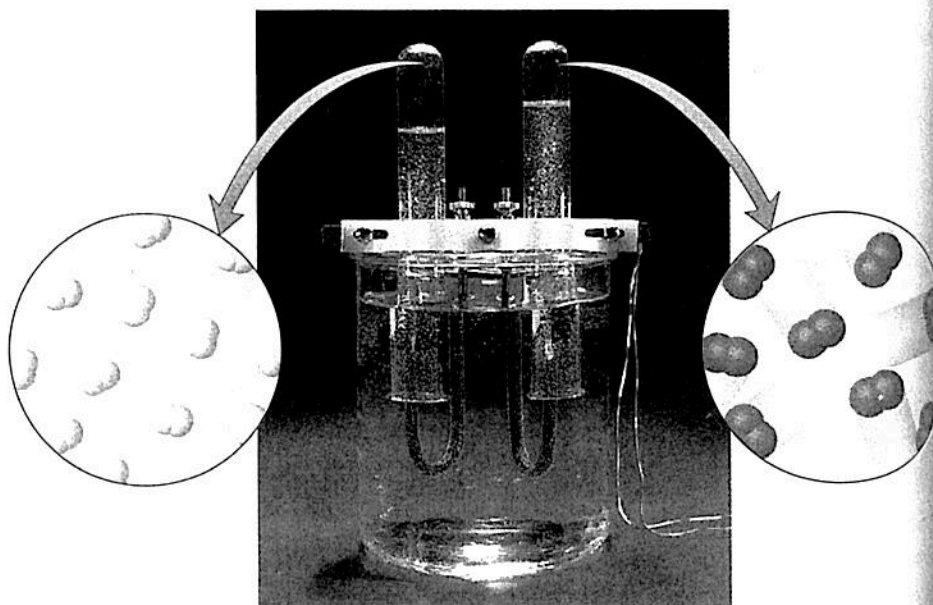


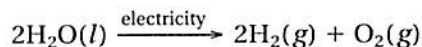
Figure 9.11

The electrolysis of water causes it to decompose into hydrogen and oxygen.



Sample Problem 9-5

How many molecules of oxygen are produced when a sample of 29.2 g of water is decomposed by electrolysis according to this balanced equation?



1. ANALYZE List the knowns and the unknown.

Knowns:

- mass of water = 29.2 g H_2O
- 2 mol H_2O = 1 mol O_2 (from balanced equation)
- 1 mol H_2O = 18.0 g H_2O (molar mass)
- 1 mol O_2 = 6.02×10^{23} molecules O_2

Unknown:

- molecules of oxygen = ? molecules O_2

Use appropriate conversion factors to convert the given quantity in grams to number of molecules.

2. CALCULATE Solve for the unknown.

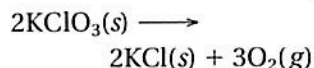
$$\begin{array}{ccccccc}
 29.2 \text{ g H}_2\text{O} & \times & \frac{1 \text{ mol H}_2\text{O}}{18.0 \text{ g H}_2\text{O}} & \times & \frac{1 \text{ mol O}_2}{2 \text{ mol H}_2\text{O}} & \times & \frac{6.02 \times 10^{23} \text{ molecules O}_2}{1 \text{ mol O}_2} \\
 \text{Given} & & \text{Change} & & \text{Mole ratio} & & \text{Change to molecules} \\
 \text{quantity} & & \text{to moles} & & & & \\
 & & & & & & \\
 & & & & & & = 4.88 \times 10^{23} \text{ molecules O}_2
 \end{array}$$

3. EVALUATE Does the result make sense?

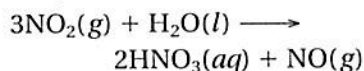
The given mass of water should produce a little less than 1 mol of oxygen, or a little less than Avogadro's number of molecules. The answer should have three significant figures.

Practice Problems

13. How many molecules of oxygen are produced by the decomposition of 6.54 g of potassium chlorate (KClO_3)?



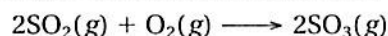
14. The last step in the production of nitric acid is the reaction of nitrogen dioxide with water.



How many grams of nitrogen dioxide must react with water to produce 5.00×10^{22} molecules of nitrogen monoxide?

Sample Problem 9-6

Assuming STP, how many liters of oxygen are needed to produce 19.8 L SO₃ according to this balanced equation?



1. ANALYZE List the knowns and the unknown.

Knowns:

- volume of sulfur trioxide = 19.8 L
- 2 mol SO₃ = 1 mol O₂ (from balanced equation)
- 1 mol SO₃ = 22.4 L SO₃ (at STP)
- 1 mol O₂ = 22.4 L O₂ (at STP)

Unknown:

- volume of oxygen = ? L O₂

2. CALCULATE Solve for the unknown.

$$19.8 \text{ L SO}_3 \times \frac{1 \text{ mol SO}_3}{22.4 \text{ L SO}_3} \times \frac{1 \text{ mol O}_2}{2 \text{ mol SO}_3} \times \frac{22.4 \text{ L O}_2}{1 \text{ mol O}_2} = 9.90 \text{ L O}_2$$

Given
Change
Mole ratio
Change
quantity
to moles

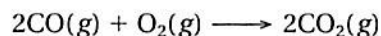
to liters

3. EVALUATE Does the result make sense?

Because 2 mol SO₃ is produced for each 1 mol O₂ that reacts, the volume of O₂ should be half the volume of SO₃. The answer should have three significant figures.

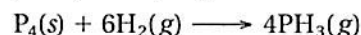
Practice Problems

- 15.** The equation for the combustion of carbon monoxide is



How many liters of oxygen are required to burn 3.86 L of carbon monoxide?

- 16.** Phosphorus and hydrogen can be combined to form phosphine (PH₃).



How many liters of phosphine are formed when 0.42 L of hydrogen reacts with phosphorus?

Chem ASAP!

Problem-Solving 16

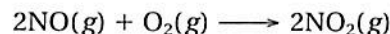
Solve Problem 16 with the help of an interactive guided tutorial.



In Sample Problem 9-6, did you notice that the 22.4 L/mol factors canceled out? This will always be true in a volume-volume problem. The coefficients in a balanced chemical equation indicate the relative numbers of moles. The coefficients also indicate the relative volumes of interacting gases. The volume can be expressed in any unit. What are some other units of volume?

Sample Problem 9-7

Nitrogen monoxide and oxygen gas combine to form the brown gas nitrogen dioxide. How many milliliters of nitrogen dioxide are produced when 3.4 mL of oxygen reacts with an excess of nitrogen monoxide? Assume conditions of STP.



1. ANALYZE List the knowns and the unknown.

Knowns:

- volume of oxygen = 3.4 mL O₂
- 1 mL O₂ = 2 mL NO₂ (from balanced equation)

Unknown:

- volume of nitrogen dioxide = ? mL NO₂

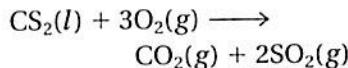


Figure 9.12

The brown gas nitrogen dioxide is a component of photochemical smog, which builds up when still air hangs over a city or other pollution source. Persistent smog can pose a health danger.

Practice Problems

Consider this equation.



17. Calculate the volume of sulfur dioxide produced when 27.9 mL O_2 reacts with carbon disulfide.
18. How many deciliters of carbon dioxide are produced when 0.38 L SO_2 is formed?

Sample Problem 9-7 (cont.)

2. **CALCULATE** Solve for the unknown.

$$3.4 \text{ mL } \text{O}_2 \times \frac{2 \text{ mL } \text{NO}_2}{1 \text{ mL } \text{O}_2} = 6.8 \text{ mL } \text{NO}_2$$

Given quantity Volume ratio

3. **EVALUATE** Does the result make sense?

The conversion is $\text{mL O}_2 \longrightarrow \text{mL NO}_2$. The given quantity of O_2 is multiplied by the volume ratio from the balanced equation that allows the given unit to cancel. Because the volume ratio is 2 volumes NO_2 to 1 volume O_2 , the calculated volume of NO_2 should be twice the given volume of O_2 . The answer should have two significant figures.

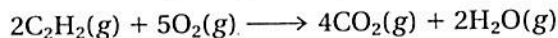
section review 9.2

19. Isopropyl alcohol ($\text{C}_3\text{H}_7\text{OH}$) burns in air according to this equation:



- a. Calculate the moles of oxygen needed to react with 3.40 mol $\text{C}_3\text{H}_7\text{OH}$.
 - b. Find the moles of each product formed when 3.40 mol $\text{C}_3\text{H}_7\text{OH}$ reacts with oxygen.
20. What ratio is used to carry out each conversion?
- a. mol CH_4 to g CH_4
 - b. L $\text{CH}_4(g)$ to mol $\text{CH}_4(g)$ (at STP)
 - c. molecules CH_4 to mol CH_4

21. The combustion of acetylene gas is represented by this equation:



- a. How many grams of CO_2 and grams of H_2O are produced when 52.0 g C_2H_2 burns?
- b. How many grams of oxygen are required to burn 52.0 g C_2H_2 ?
- c. Use the answers from a and b to show that this equation obeys the law of conservation of mass.

22. Tin(II) fluoride, formerly found in many kinds of toothpaste, is formed in this reaction:



- a. How many liters of HF are needed to produce 9.40 L H_2 at STP?
- b. How many molecules of H_2 are produced by the reaction of tin with 20.0 L HF at STP?
- c. How many grams of SnF_2 can be made by reacting 7.42×10^{24} molecules of HF with tin?



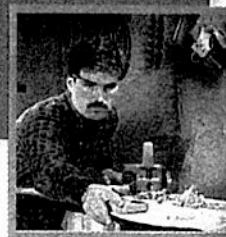
portfolio project

Using the Internet or the library, research analytical balances. How small a mass or large a mass can these types of balances measure? Compare different balances to find out whether there is a relationship between precision and cost.



Chem ASAP! Assessment 9.2 Check your understanding of the important ideas and concepts in Section 9.2.

LIMITING REAGENT AND PERCENT YIELD



objectives

- ▶ Identify and use the limiting reagent in a reaction to calculate the maximum amount of product(s) produced and the amount of excess reagent
- ▶ Calculate theoretical yield, actual yield, or percent yield given appropriate information

key terms

- ▶ limiting reagent
- ▶ excess reagent
- ▶ theoretical yield
- ▶ actual yield
- ▶ percent yield

If a carpenter had two table tops and seven table legs, he would have difficulty building more than one functional four-legged table. The first table would require four of the legs, leaving just three legs for the second table. In this case, the number of table legs is the limiting factor in the construction of four-legged tables. A similar concept applies in chemistry when knowing the exact amounts of reactants and products in a chemical reaction is crucial. How does a limiting reagent affect a chemical reaction?

What Is a Limiting Reagent?

Perhaps you know from your own experience that many cooks follow a recipe when making a new dish. They know that sufficient quantities of all the ingredients must be available. Suppose, for example, that you are preparing to make lasagna and you have more than enough meat, tomato sauce, ricotta cheese, eggs, mozzarella cheese, spinach, and seasoning on hand. However, you have only half a box of lasagna noodles. The amount of lasagna you can make will be limited by the amount of noodles you have. Thus the noodles are the limiting reagent in this baking venture. Figure 9.13 illustrates another example of a limiting reagent in the kitchen. A chemist often faces a similar situation. It is impossible for a chemist to make a certain amount of a desired compound if there is an insufficient quantity of any of the required reactants.

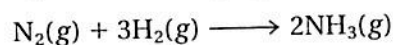
As you know, a balanced chemical equation is a chemist's recipe—a recipe that can be interpreted on a microscopic scale (interacting particles) or on a macroscopic scale (interacting moles). The coefficients used to write



Figure 9.13

The amount of product is determined by the quantity of the limiting reagent. In this example, the rolls are the limiting reagent. No matter how much of the other ingredients you have, with two rolls you can make only two sandwiches.

the balanced equation give both the ratio of representative particles and the mole ratio. Recall the equation for the preparation of ammonia.



When one molecule (mole) of N_2 reacts with three molecules (moles) of H_2 , two molecules (moles) of NH_3 are produced. What would happen if two molecules (moles) of N_2 reacted with three molecules (moles) of H_2 ? Would more than two molecules (moles) of NH_3 be formed? Figure 9.14 shows both the particle and the mole interpretations of this problem.

Before the reaction takes place, nitrogen and hydrogen are present in a 2:3 molecule (mole) ratio. The reaction takes place according to the balanced equation. One molecule (mole) of N_2 reacts with three molecules (moles) of H_2 to produce two molecules (moles) of NH_3 . At this point, all the hydrogen has been used up, and the reaction stops. One molecule (mole) of unreacted nitrogen is left in addition to the two molecules (moles) of NH_3 that have been produced by the reaction.

In this reaction, only the hydrogen is completely used up. It is called the **limiting reagent**. As the name implies, the **limiting reagent** limits or determines the amount of product that can be formed in a reaction. The reaction occurs only until the limiting reagent is used up. By contrast, the reactant that is not completely used up in a reaction is called the **excess reagent**. In this example, nitrogen is the excess reagent because some nitrogen will remain unreacted. You probably know that if you put a glass over a burning candle, the candle goes out. In this example of the combustion of candle wax, what is the limiting reagent?

Sometimes in problems, the given quantities of reactants are expressed in units other than moles. In such cases, the first step in the solution is to convert each reactant to moles. Then the limiting reagent can be identified, as in Sample Problem 9-8. Finally, the amount of product can be determined from the given amount of limiting reagent.

Chem ASAP!

Animation 8

Apply the limiting reagent concept to the production of iron from iron ore.



Chemical Equations			
	$\text{N}_2(\text{g})$	+	$3\text{H}_2(\text{g}) \longrightarrow 2\text{NH}_3(\text{g})$
"Microscopic recipe"	1 molecule N_2	+	3 molecules $\text{H}_2 \longrightarrow 2$ molecules NH_3
"Macroscopic recipe"	1 mol N_2	+	3 mol $\text{H}_2 \longrightarrow 2$ mol NH_3




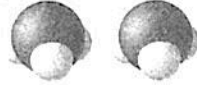
Experimental Conditions			
	Reactants		Products
Before reaction	 2 molecules N_2	 3 molecules H_2	0 molecules NH_3
After reaction	 1 molecule N_2	0 molecules H_2	 2 molecules NH_3

Figure 9.14

The "recipe" calls for 3 molecules of H_2 for every 1 molecule of N_2 . In this particular experiment, H_2 is the limiting reagent and N_2 is in excess. Would the amount of products formed change if you started with 4 molecules of N_2 and 3 molecules of H_2 ?

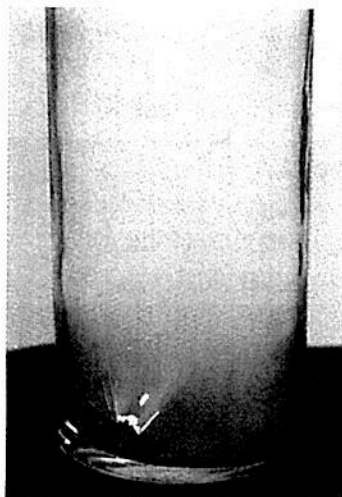
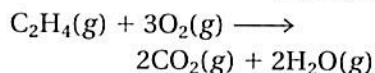


Figure 9.15

Igniting sodium metal in chlorine gas produces a white smoke of NaCl and bright yellow light.

Practice Problems

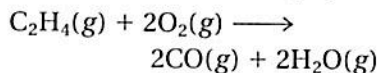
23. The equation for the complete combustion of ethene (C_2H_4) is



If 2.70 mol C_2H_4 is reacted with 6.30 mol O_2 ,

- identify the limiting reagent.
- calculate the moles of water produced.

24. The equation for the incomplete combustion of ethene (C_2H_4) is



If 2.70 mol C_2H_4 is reacted with 6.30 mol O_2 ,

- identify the limiting reagent.
- calculate the moles of water produced.

Chem ASAP!

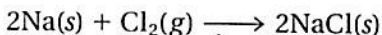
Problem-Solving 24

Solve Problem 24 with the help of an interactive guided tutorial.



Sample Problem 9-8

Sodium chloride can be prepared by the reaction of sodium metal with chlorine gas.



Suppose that 6.70 mol Na reacts with 3.20 mol Cl_2 .

- What is the limiting reagent?
- How many moles of NaCl are produced?

1. **ANALYZE** List the knowns and the unknown for a.

Knowns:

- moles sodium = 6.70 mol Na
- moles chlorine = 3.20 mol Cl_2
- 2 mol Na = 1 mol Cl_2 (from balanced equation)

Unknown:

- limiting reagent = ?

The known amount of one of the reactants is multiplied by the mole ratio from the balanced equation to calculate the required amount of the other reactant. Sodium is chosen arbitrarily here: mol Na \rightarrow mol Cl_2 .

2. **CALCULATE** Solve for the unknown.

$$6.70 \text{ mol Na} \times \frac{1 \text{ mol Cl}_2}{2 \text{ mol Na}} = 3.35 \text{ mol Cl}_2$$

Given
amount

Mole
ratio

Required
amount

This calculation indicates that 3.35 mol Cl_2 is needed to react with 6.70 mol Na. Because only 3.20 mol Cl_2 is available, however, chlorine becomes the limiting reagent. Sodium, then, must be in excess.

1. **ANALYZE** List the knowns and the unknown for b.

Knowns:

- amount of limiting reagent = 3.20 mol Cl_2
- 1 mol Cl_2 = 2 mol NaCl (from balanced equation)

Unknown:

- moles of sodium chloride = ? mol NaCl

2. **CALCULATE** Solve for the unknown.

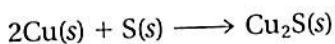
$$3.20 \text{ mol Cl}_2 \times \frac{2 \text{ mol NaCl}}{1 \text{ mol Cl}_2} = 6.40 \text{ mol NaCl}$$

3. **EVALUATE** Do the results make sense?

Because the ratio of the given moles of sodium to chlorine was greater than 2:1, which is the ratio from the balanced equation, sodium should be in excess and chlorine should be the limiting reagent.

Sample Problem 9-9

As illustrated in Figure 9.16, the properties of copper(I) sulfide are very different from the properties of the elements copper and sulfur.



- What is the limiting reagent when 80.0 g Cu reacts with 25.0 g S?
- What is the maximum number of grams of Cu_2S that can be formed?

1. ANALYZE List the knowns and the unknown for a.

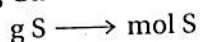
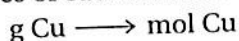
Knowns:

- mass of copper = 80.0 g Cu
- mass of sulfur = 25.0 g S
- 2 mol Cu = 1 mol S
(from balanced equation)

Unknown:

- limiting reagent = ?

The number of moles of each reactant must first be found:



The balanced equation is used to calculate the number of moles of one reactant needed to react with the given amount of the other reactant:



2. CALCULATE Solve for the unknown.

$$80.0 \text{ g Cu} \times \frac{1 \text{ mol Cu}}{63.5 \text{ g Cu}} = 1.26 \text{ mol Cu}$$

$$25.0 \text{ g S} \times \frac{1 \text{ mol S}}{32.1 \text{ g S}} = 0.779 \text{ mol S}$$

$$1.26 \text{ mol Cu} \times \frac{1 \text{ mol S}}{2 \text{ mol Cu}} = 0.630 \text{ mol S}$$

Given quantity	Mole ratio	Needed amount
----------------	------------	---------------

Comparing the amount of sulfur needed (0.630 mol S) with the given amount (0.779 mol S) indicates that sulfur is in excess. Thus copper is the limiting reagent.

1. ANALYZE List the knowns and the unknown for b.

Knowns:

- limiting reagent = 1.26 mol Cu
- 2 mol Cu = 1 mol Cu_2S (from balanced equation)
- 1 mol Cu_2S = 159.1 g Cu_2S (molar mass)

Unknown:

- mass copper(I) sulfide = ? g Cu_2S

The limiting reagent, which was determined in the last step, is used to calculate the maximum amount of Cu_2S formed:

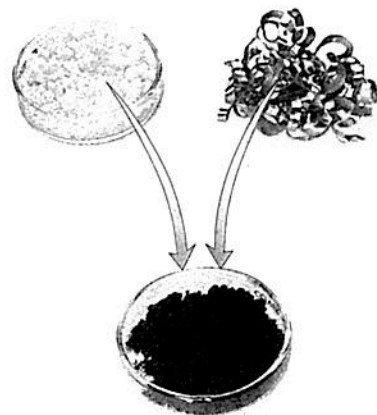
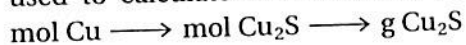
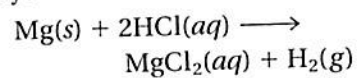


Figure 9.16

Black crystalline copper(I) sulfide (bottom) is formed as a product when the reactants, sulfur (top left) and copper (top right), are heated together.

Practice Problems

25. Hydrogen gas can be produced in the laboratory by the reaction of magnesium metal with hydrochloric acid.

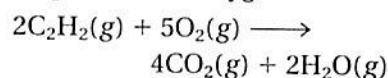


- Identify the limiting reagent when 6.00 g HCl reacts with 5.00 g Mg.
- How many grams of hydrogen can be produced when 6.00 g HCl is added to 5.00 g Mg?

Sample Problem 9-9 (cont.)

Practice Problems (cont.)

26. Acetylene (C_2H_2) will burn in the presence of oxygen.



How many grams of water can be produced by the reaction of 2.40 mol C_2H_2 with 7.4 mol O_2 ?

2. **CALCULATE** Solve for the unknown.

$$1.26 \text{ mol-Cu} \times \frac{1 \text{ mol-Cu}_2\text{S}}{2 \text{ mol-Cu}} \times \frac{159.1 \text{ g Cu}_2\text{S}}{1 \text{ mol-Cu}_2\text{S}} = 1.00 \times 10^2 \text{ g Cu}_2\text{S}$$

The given quantity of copper, 80.0 g, could have been used for this step instead of the moles of copper, which were calculated in the very first step of the solution.

3. **EVALUATE** Do the results make sense?

Copper is the limiting reagent in this reaction. The maximum number of grams of Cu_2S produced should be more than the amount of copper that initially reacted because copper is combining with sulfur. The amount of Cu_2S produced should be less than the sum of copper and sulfur ($80.0 \text{ g Cu} + 25.0 \text{ g S} = 105.0 \text{ g}$) because sulfur was in excess.

Calculating the Percent Yield

In theory, when a teacher gives an exam to the class, every student should get a grade of 100%. For a variety of reasons, this generally does not occur. Instead, the performance of the class is usually spread over a range of grades. Your exam grade, expressed as a percentage, is a quantity that shows how well you did on the exam (questions answered correctly) compared with how well you could have done if you had answered all the questions correctly (100%). This calculation is analogous to the percent yield calculation that you do in the laboratory when the product from a chemical reaction is less than you expected based on the balanced chemical equation.

So far in this chapter, you have probably assumed that when doing stoichiometric problems things do not go wrong in chemical reactions. This assumption is as faulty as assuming that all students will score 100% on an exam. When an equation is used to calculate the amount of product that will form during a reaction, a value representing the theoretical yield is obtained. The **theoretical yield** is the maximum amount of product that could be formed from given amounts of reactants. In contrast, the amount of product that actually forms when the reaction is carried out in the laboratory is called the **actual yield**. The actual yield is often less than the theoretical yield. The **percent yield** is the ratio of the actual yield to the theoretical yield expressed as a percent. The percent yield measures the efficiency of the reaction.

$$\text{Percent yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100\%$$

A percent yield should not normally be larger than 100%. Many factors cause percent yields to be less than 100%. Reactions do not always go to completion; when this occurs, less than the calculated amount of product is formed. Impure reactants and competing side reactions may cause unwanted products to form. Actual yield can also be lower than the theoretical yield due to a loss of product during filtration or in transferring

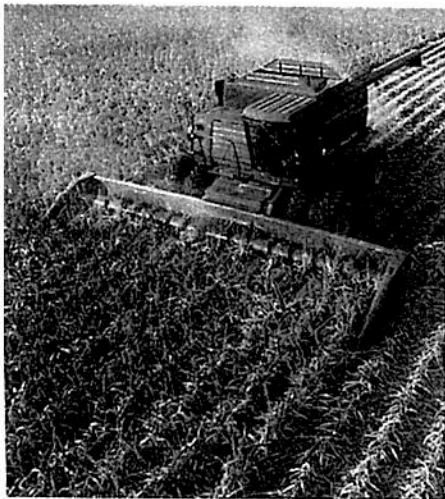
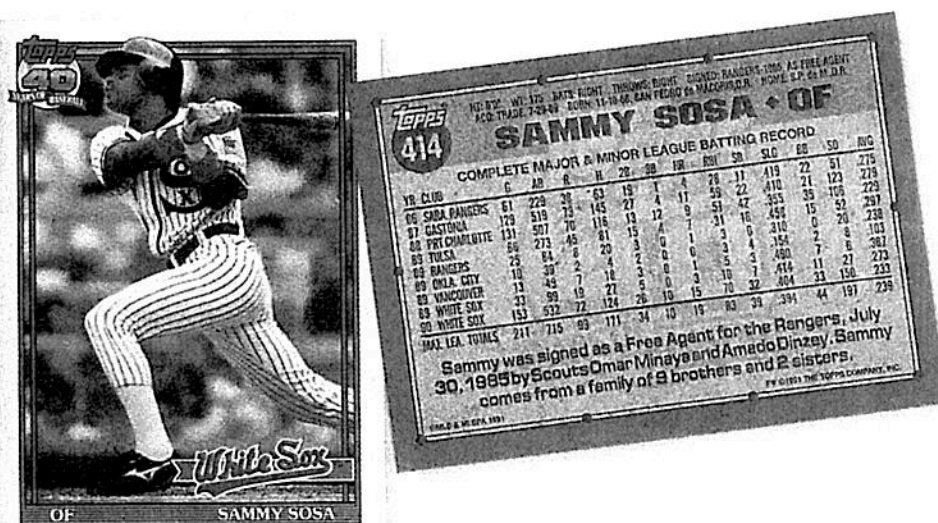


Figure 9.17

The productivity of a farm is measured in yield. Because growing conditions may vary from year to year, the actual yield often differs from the theoretical yield.



between containers. Moreover, if reactants or products have not been carefully measured, a percent yield of 100% is unlikely.

An actual yield is an experimental value. Figure 9.19 shows a typical laboratory procedure for determining the actual yield of a product of a decomposition reaction. If you do not do an experiment, you cannot calculate a percent yield unless you are given the value of an actual yield. For reactions in which percent yields have been determined, you can calculate and therefore predict an actual yield if the reaction conditions remain the same. A farmer's crop yield could also be expressed as a percent yield. What factors would a farmer use to predict a theoretical yield?

Figure 9.19

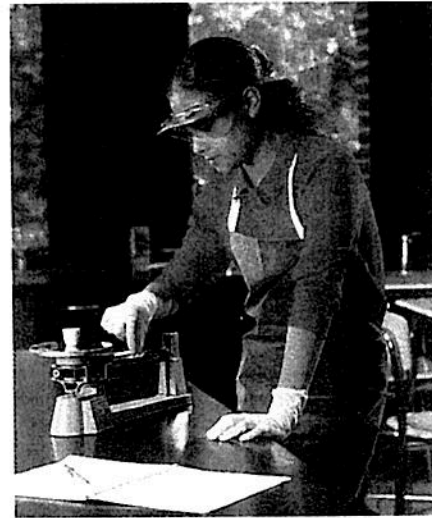
- (a) The mass of sodium hydrogen carbonate (NaHCO_3), the reactant, is measured.
 (b) The reactant is heated. (c) The mass of one of the products, sodium carbonate (Na_2CO_3), the actual yield, is measured after the reaction is completed. The percent yield can be calculated once the actual yield has been determined. What are the other products of this reaction?



(a)



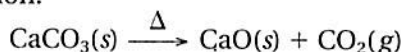
(b)



(c)

Sample Problem 9-10

Calcium carbonate is decomposed by heating, as shown in the following equation.



- What is the theoretical yield of CaO if 24.8 g CaCO₃ is heated?
- What is the percent yield if 13.1 g CaO is produced?

1. ANALYZE List the knowns and the unknown for a.

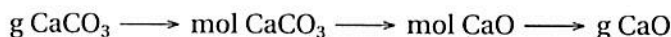
Knowns:

- mass of calcium carbonate = 24.8 g CaCO₃
- 1 mol CaCO₃ = 1 mol CaO (from balanced equation)
- 1 mol CaCO₃ = 100.1 g CaCO₃ (molar mass)
- 1 mol CaO = 56.1 g CaO (molar mass)

Unknown:

- theoretical yield of calcium oxide = ? g CaO

The theoretical yield can be calculated using the mass of the reactant:



2. CALCULATE Solve for the unknown.

First, the theoretical yield of the reaction is calculated.

$$24.8 \text{ g CaCO}_3 \times \frac{1 \text{ mol CaCO}_3}{100.1 \text{ g CaCO}_3} \times \frac{1 \text{ mol CaO}}{1 \text{ mol CaCO}_3} \times \frac{56.1 \text{ g CaO}}{1 \text{ mol CaO}} = 13.9 \text{ g CaO}$$

1. ANALYZE List the knowns and the unknown for b.

Knowns:

- actual yield = 13.1 g CaO
- theoretical yield = 13.9 g CaO (from a)
- percent yield = $\frac{\text{actual yield}}{\text{theoretical yield}} \times 100\%$

Unknown:

- percent yield = ? %

2. CALCULATE Solve for the unknown.

$$\text{percent yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100\%$$

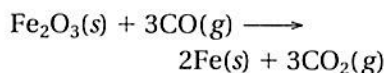
$$\text{percent yield} = \frac{13.1 \text{ g CaO}}{13.9 \text{ g CaO}} \times 100\% = 94.2\%$$

3. EVALUATE Does the result make sense?

In this example, the actual yield is slightly less than the theoretical yield. Therefore, the percent yield should be slightly less than 100%. The answer should have three significant figures.

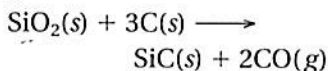
Practice Problems

27. When 84.8 g of iron(III) oxide reacts with an excess of carbon monoxide, 54.3 g of iron is produced.



What is the percent yield of this reaction?

28. If 50.0 g of silicon dioxide is heated with an excess of carbon, 27.9 g of silicon carbide is produced.



What is the percent yield of this reaction?

Chem ASAP!

Problem-Solving 28

Solve Problem 28 with the help of an interactive guided tutorial.



MINI LAB



Limiting Reagents

PURPOSE

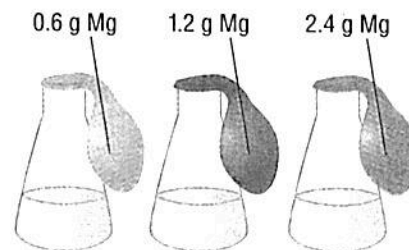
To illustrate the concept of a limiting reagent in a chemical reaction.

MATERIALS

- graduated cylinder
- balance
- 3 250-mL Erlenmeyer flasks
- 3 rubber balloons
- 4.2 g magnesium ribbon
- 300 mL 1.0M hydrochloric acid

PROCEDURE

1. Add 100 mL of the hydrochloric acid solution to each flask.
2. Weigh out 0.6 g, 1.2 g, and 2.4 g of magnesium ribbon, and place each sample into its own balloon.
3. Stretch the end of each balloon over the mouth of each flask. Do not allow the magnesium ribbon in the balloon to fall into the flask.
4. Magnesium reacts with hydrochloric acid to form hydrogen gas. When you mix the magnesium with the hydrochloric acid in the next step, you will generate a certain volume of hydrogen gas. How do you think the volume of hydrogen produced in each flask will compare?
5. Lift up on each balloon and shake the magnesium metal down into each flask. Observe the volume of gas produced until the reaction in each flask is completed.

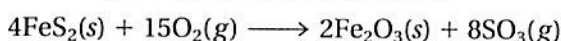


ANALYSIS AND CONCLUSIONS

1. How did the volumes of hydrogen gas produced, as measured by the size of the balloons, compare? Did the results agree with your prediction?
2. Write a balanced equation for the reaction between magnesium metal and hydrochloric acid.
3. The 100 mL of hydrochloric acid contained 0.10 mol HCl. Show by calculation why the balloon with 1.2 g Mg inflated to about twice the size of the balloon with 0.60 g Mg.
4. Show by calculation why the balloons with 1.2 g and 2.4 g Mg inflated to approximately the same volume. What was the limiting reagent when 2.4 g Mg was added to the acid?

section review 9.3

29. What is a limiting reagent? An excess reagent?
30. What is the percent yield if 4.65 g of copper is produced when 1.87 g of aluminum reacts with an excess of copper(II) sulfate?
31. What is the difference between an actual yield and a theoretical yield? Which yield is larger for a given reaction? How are these values used to determine percent yield?
32. How many grams of SO_3 are produced when 20.0 g FeS_2 reacts with 16.0 g O_2 according to this balanced equation?



Chem ASAP! Assessment 9.3 Check your understanding of the important ideas and concepts in Section 9.3.